

EQUATIONS OF MOTION

Newton's Method

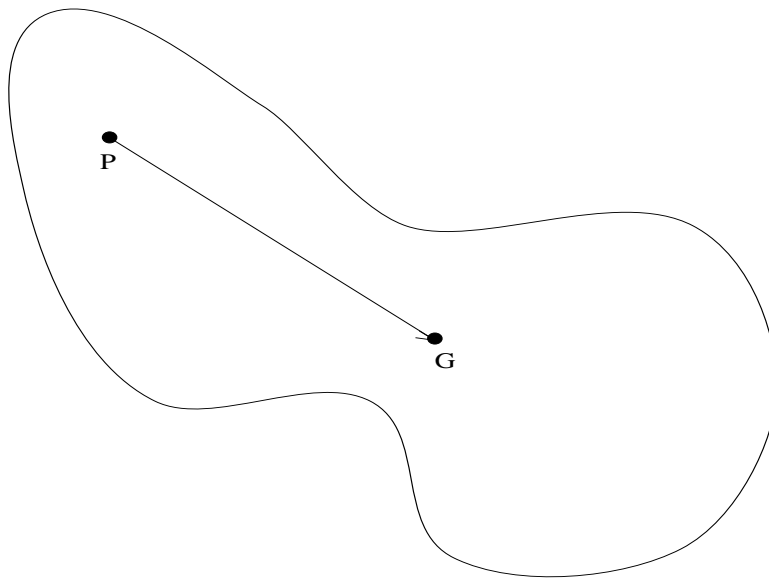
Force Balance:

$$\sum \overline{Forces} = M \overline{\ddot{q}_G}$$

Moment Balance:

$$\sum_P \overline{Moments} = J_P \overline{\ddot{\theta}} + \overline{r_{G/P}} \times M \overline{\ddot{q}_P}$$

$$\sum_P \overline{Moments} = J_G \overline{\ddot{\theta}} + \overline{r_{G/P}} \times M \overline{\ddot{q}_G}$$



EQUATIONS OF MOTION

d'Alembert's Method

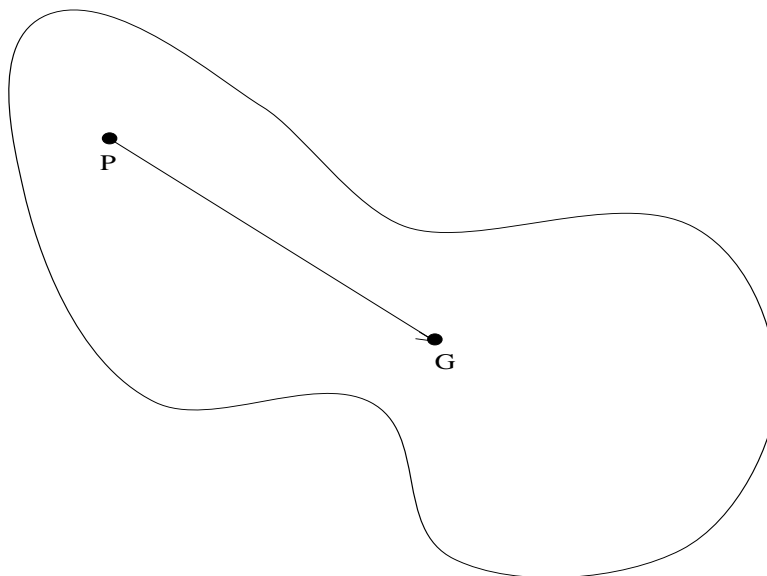
Force Balance:

$$\sum \overline{Forces} - M \overline{\ddot{q}} = 0$$

Moment Balance:

$$\sum_P \overline{Moments} - J_P \overline{\ddot{\theta}} - \overline{r_{G/P}} \times M \overline{\ddot{q}_P} = 0$$

$$\sum_P \overline{Moments} - J_G \overline{\ddot{\theta}} - \overline{r_{G/P}} \times M \overline{\ddot{q}_G} = 0$$



EQUATIONS OF MOTION

Newton's Method (d'Alembert's Method)

Typical problems with Newton/d'Alembert formulation:

- **Be sure to establish the number of degrees of freedom first and formulate all terms in only those variables. Clearly identify which degrees of freedom are relative coordinates versus absolute coordinates. Also, clearly identify what will be the positive direction of motion for each coordinate. Watch out for rotational/translational problems. State any constraint relationships that relate independent and dependent coordinates.**
- **Evaluate the static balance for the problem in order to determine whether the orientation of the system in the gravity field will effect the equations of motion (Are the weights of the objects balanced by an initial static deflection in the springs?). When in doubt, perform a static force balance to determine the appropriate constraint equation.**
- **For displacement, velocity and acceleration terms, be sure to develop *absolute or relative* displacement, velocity and accelerations of appropriate points as required. Watch out for 2-D and 3-D vector motions.**

EQUATIONS OF MOTION

Newton's Method (d'Alembert's Method)

Typical problems with Newton/d'Alembert formulation:

- **Be sure to draw the appropriate free body diagrams for each mass (or combination of masses) in the system.**
 - **Whenever the system is separated in order to draw a free body diagram, replace the separation with the appropriate internal forces/moments (equal and opposite forces/moments on each side of the separation).**
 - **Do not move forces/moments arbitrarily from one mass to another. The internal forces account for the effects of one mass on another.**
- **Develop one equation of motion for each degree of freedom of the system using Newton's or d'Alembert's method. Be sure to watch for moving reference frame issues. Also, check that the units are the same for each term in an equation (Forces + Moments: NOT!)**
- **If necessary, once the exact equations of motion have been determined, linearize the equations of motion by neglecting nonlinear terms in the equations of motion. Note that the linear equations of motion may not adequately describe the original equations of motion if some of the terms that have been neglected are not insignificant.**

