Identifying Nonlinear Parameters for Reduced Order Models. Part II, Validation using Experimental Data

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Abstract

Assembling nonlinear dynamic models of structures is the goal of numerous research and development organizations. Such a predictive capability is required in the development of advanced, high-performance aircraft structures. Specifically, the ability to predict the response of complex structures to aero-acoustic loading has long been a United States Air Force goal. Sonic fatigue has plagued the Air Force since the advent and adoption of the aircraft turbine engine. While the problem has historically been a maintenance one, predicting dynamic response is crucial for future aerospace vehicles. Decades have been spent investigating the dynamic response and untimely failures of aircraft structures, yet little work has been accomplished towards developing practical nonlinear prediction methods. The aim of this paper and a companion one is to present a novel means of assembling nonlinear reduced order models using experimental data and an analytical basis. The companion paper, Part I, outlines a unique extension of a recently introduced nonlinear identification method; Nonlinear Identification through Feedback of the Outputs (NIFO). This paper, Part II, details a high-fidelity experiment and the resulting successful identification conducted on a well characterized clamped-clamped beam subjected to broadband random loading. Geometric nonlinear parameters were identified for a multiple degree-of-freedom (MDOF) nonlinear reduced order model. The assembled MDOF nonlinear model was used to successfully predict the experimental response of the beam to another loading condition. Beam response spectra and displacements from the prediction model also compare well with the experimental results.

1. Nomenclature

\[ f(t) \] Time Domain Forcing Function

\[ \phi \] Basis Vector

\[ t, \omega \] Time, Frequency

\[ x(t) \] Displacement in Physical Space

\[ \tilde{A}_r \] Scaled Nonlinear Coefficient

\[ \tilde{F}[\cdot] \] Fourier Transform

\[ H(\omega) \] Linear Modal Frequency Response Function

\[ X(\omega) \] Physical Response in the Frequency Domain

2. Introduction

The USAF is interested in developing a nonlinear dynamic prediction capability. This capability is required in order to predict the response of structures expected to survive in extreme acoustic and thermal environments. There are numerous analytical and experimental based reduced order identification methods currently under development by various US Government and university research groups. The modal NIFO method introduced in Part I of this paper is an experimental one, considered complimentary to the developing analytical reduced order prediction tools. Part I of the paper provided an introduction of the method via a comparison with an analytical model. In Part II, the modal NIFO method is exercised via experimental data. Experimental data is used to identify and assemble mathematical prediction models. Having an experimental based tool such as this one is
useful, enabling researchers to consider the assumptions that arise during FE modeling and the resulting reduced order models, namely boundary conditions, material properties and the nonlinearities limited to the FE elements of choice. Specifically, by identifying reduced order models from experimental data, researchers are assembling models based upon the actual model and not an idealized one. Of course, the downside is that laboratory experiments tend to be noisy, introducing error and leading to difficulties in accurately identifying nonlinear parameters. Further, as compared with the analytical methods, the modal NIFO method presented herein provides extensive temporal information but relatively limited spatial information. The emphasis of this proposed tool is therefore on its complimentary nature and not one of supplanting any analytical tool discussed in Part I.

A survey of the purely experimental based modal identification methods will now be presented. Thouverez and Jezequel [1] present a time-domain modal NARMAX (Nonlinear Auto Regressive Moving Average with eXogenous outputs) approach. There are several interesting characteristics of their method worth noting. The authors discuss the fact that nonlinear localization is lost when the problem is transformed into reduced order space. The authors also mention that a 'pre-selection' method further reduces the number of terms in the low order equations, but this presupposition requires accurate nonlinear characterization of the structure of interest. Finally, the authors use an eigenvector updating procedure both to ensure that the identified linear terms are indeed uncoupled as expected for a modal based method and to appropriately correct the eigenvectors used in the transformation.

Another approach to nonlinear parameter identification for reduced order models is the one taken by Yasuda and Kamiya [2]. Yasuda begins with an elasticity approach and arrives at the governing modal equations nonlinear in stiffness via the Galerkin Method. This proposed time-domain approach is separated into two complimentary methods. The first being to take measured response data and solve explicitly for the basis set via a weighted eigenvalue problem. Then, linear and nonlinear modal parameters are identified using a least squares approach. Their second approach is to simultaneously identify all parameters including the basis set. This is accomplished by the minimization of an energy expression requiring an iterative scheme. Initial conditions for this iterative step are obtained from the results of the first method. Good agreement was obtained between analysis and experiment using the second method detailed in the reference.

Recently, Platten et al. [3,4] and Naylor et al. [5] applied their Nonlinear Resonant Decay Method (NL-RDM) to identify modal models for both analytical and experimental systems. The NL-RDM method is also a multi-step process accomplished in the time domain. First, a general sense of the linear and nonlinear characteristics of the system in question is obtained through a series of tests conducted at various input levels. Second, the modal properties of the nominal linear system are obtained via traditional modal testing methods. Of prime importance in this step is the estimation of the normal mode shapes or the basis intended for the transformation between physical and modal space. Next, the system is excited via a burst sine excitation on a mode-by-mode basis. The excitation is applied at a level appropriate enough to exercise the nonlinearities of the respective mode. The nonlinear parameters are then identified, again on a mode-by-mode basis. Finally, a full nonlinear modal model is assembled. One of the assumptions in the methodology is the subjective truncation of any mode that does not appear to interact or have influence over other modes. While this assumption is in keeping with the idea of linear modal truncation, it should be noted that the response of a nonlinear system is a function but not necessarily a linear superposition of all of the structural modes. This is particularly important for nonlinear modal models, for while a mode may not be perceived to contribute to the nonlinear aspects of the response at particular loading inputs, the issue of modal saturation can play a significant role in nonlinear systems [6].

In an extensive dissertation, Siller [7] recently discussed, among other things a direct nonlinear identification approach similar to the work proposed herein. The method presented by Siller is a reduced order one, and also accomplished in the frequency domain. There are however some limitations with Siller's work. The nonlinear parameters are necessarily constant as averages are taken on a frequency basis. The modal NIFO method presented herein is not constrained by this assumption since the nonlinear coefficients in modal space are identified frequency-by-frequency. Further, Siller's nonlinear modal parameter identification method relies on a multi-step identification approach, where the measured response quantities are first separated into linear measured and nonlinear unmeasured quantities. Second, using the results of a preliminary experimental modal analysis, the linear portion of the assumed model form is removed from the nonlinear part. Finally, an assumed nonlinear form in physical space is then used to identify the constant nonlinear coefficients. The physical space identification formulation allows researchers to identify the location of any nonlinearities. A final distinction between Siller's work and this proposed work is the comparison with experimental work and the proposed characterization useful for determining both the type of nonlinearities present as well as the coupling between
assumed modes. There is no single comparison between an experimental based nonlinear modal identification method and a well characterized aircraft type structural experiment exhibiting this form of response.

Masri et. al. [8] present a two-stage time-domain modal approach. In the first stage, the linear system terms are identified in a least-squares sense via expressions of the system accelerations. In the second stage, the assumed nonlinear terms, expressed as unknown nonlinear forces, are then identified. The interesting facet of the research is that in lieu of identifying a full suite of nonlinear terms, the authors recognize the utility of transforming (or ‘rotating’) the system into reduced order space via the linear system eigenvectors. Thus, the identification is accomplished. The basis selected for the rotation is a linear combination of the system displacements, velocities and forces, similar to the method of Yasuda [2]. Given the form of the assumed basis, it appears that the method requires knowledge of all DOF displacements, velocities and forces in order to adequately identify the unknown nonlinear terms. A good comparison is made between the proposed method and a 3-DOF discrete system, but no comparison with experiments is presented.

Chong and Imregun [9] present a frequency domain nonlinear modal identification method. The method assumes well-separated modes, where a particular mode of interest is not impacted by neighboring modes. Further, an iterative approach is utilized to converge to a ‘nonlinear’ mode shape that uncouples the measured response. These ‘nonlinear’ modes are assumed to consist of the respective mode of interest plus the contributions or combinations of the other well-separated modes. The intent of the method is to obtain modal parameters as functions of modal amplitudes. This underscores a significant difference with the proposed modal NIFO method, namely that the nonlinear modal equations are assumed to be coupled through the nonlinear modal expansion terms, themselves combinations of the linear mode shapes. Further, there is no restriction on the relative separation of the modes.

3. Experimental Investigation:

A high-fidelity experiment was conducted using a configuration similar to that presented by Gordon et. al. [10]. The test setup for the current effort is presented in Fig. 1.

Figure 1: Clamped-clamped beam experimental setup.

Notice that only half of the beam was instrumented, and issue to be addressed subsequently. As previously discussed in Part I, this particular experimental setup was designed to exhibit the type of nonlinear response typical of aircraft structures experiencing sonic fatigue. The nominal beam dimensions and material properties are referenced in Part I. Inertial loading was applied to the beam using a 1200-lb shaker with the shaker oriented in such a way as to mitigate the influence of gravity on the beam. In order to maintain the desired spectral content and RMS input level, a closed-loop controller was used via an accelerometer mounted to the shaker head. There were six measurement locations along the beam; acceleration was recorded at five locations using micro-accelerometers and a laser vibrometer was used to record displacement at the beam midpoint. The accelerometer measurements were integrated twice in the frequency domain to obtain the respective
displacement time histories. Throughout the testing, a sampling rate of 4096 Hz was used, and test record lengths of 48 seconds were recorded. Just as in the analytical experiment, a broadband random input between 20 and 500 Hz was used for all of the loading scenarios. This loading spectrum allowed for the first two bending modes of the beam to be excited, and is a spectrum typical of that experienced by aircraft structures in the presence of engine induced random loading.

The NIFO method of Adams and Allemang [11,12] is an elegant means of identifying nonlinear structural parameters as well as the underlying, or nominally linear parameters in a single analysis step. The final equation introduced in Part I of this paper is presented here again:

\[
\{X_i(\omega)\} = \begin{bmatrix}
H_1(\omega) & H_1(\omega)\hat{A}_{1,1,1} & H_1(\omega)\hat{A}_{1,2,2} & \ldots \\
\ldots H_1(\omega)\hat{A}_{1,1,1} & H_1(\omega)\hat{A}_{1,2,2} & \ldots 
\end{bmatrix}
\]

where recall, the unknown nonlinear coefficients are scaled by the appropriate basis set components. Note, \(X_i(\omega)\) in Eq. (1) is not the physical response of the DOF 1, but the physical contribution of mode 1 to the total response. Since experimental data is recorded in the physical domain, it will be necessary to transform or filter the data. This transformation is accomplished via the following expression:

\[
\{p(t)\} = [\phi^T \{\phi\}]^{-1} \phi^T \{x(t)\} = [\phi^T \{x(t)\}]^T
\]

where \(\phi^T\) represents the Moore-Penrose pseudo-inverse of the proposed basis set, providing a best-fit solution in a least squares sense.

### 3.1 Experimental MDOF Nonlinear Parameter Estimation

The real benefit of the proposed nonlinear identification approach, in addition to its ease of use and flexibility in assuming various nonlinear forms, is the extraction of multiple nonlinear parameters at a single time. While numerous nonlinear identification algorithms currently exist as previously discussed, the same cannot be said regarding a robust and proven MDOF reduced order method. The first two symmetric beam bending modes were used to filter the raw experimental test data. In order to accomplish this filtering, the data must be synchronous or recorded at the same time. The use of synchronous data allows for accurate capturing of the relative motion of the sensors. The sensor locations indicated by the asterisks and their position relative to the basis set are displayed in Fig. 3. The results of the successful filtering using the two symmetric normal modes displayed in Fig. 3 are now displayed in Fig. 4.
Figure 2: Symmetric beam bending modes and measurement locations used for modal filtering. (solid line: 1st symmetric bending mode; bold dashed line: 2nd symmetric bending mode; asterisks denote experimental sensor measurement locations).

Figure 3: Modal filtering of experimental 4g RMS test case: (a) Representative modal time history, (b) Filtered modal spectra. (solid line: Experimental time history filtered via 1st mode; bold dashed line: Experimental time history filtered via 2nd mode).
Note the significant difference in amplitude between the two filtered modal time histories. Further, note the differences in the spectral content, particularly at the first mode. The ability of the proposed modal NIFO method to identify a series of nonlinear coefficients along with the nominal linear modal FRFs will be demonstrated to be quite powerful. One other issue currently being investigated is the most appropriate spatial discretization, as well as the most appropriate number of sensors in order to achieve optimal filtering.

Applying the modal filtering called out in Eq. (2) using the first two symmetric beam modes and conducting the MDOF identification called out in Eq. (1), resulted in the two nominally linear FRFs and the eight nonlinear cubic parameters displayed graphically in Figs. 5 and 6. It should be noted that the values displayed in Figs. 5 and 6 represent the real parts of the nonlinear coefficients. The imaginary components were minimal as compared to the respective real components and therefore not included in the results. Also note that the identified coefficients are relatively constant functions of frequency, particularly the $A_1^{1}(1,1,1)$ and $A_1^{2}(1,1,1)$. Note the good approximation of the nominal linear modal FRFs, particularly for the second mode, in the vicinity of the respective modes.

![Experimental Modal FRF and Nonlinear Coefficients](image)

**Figure 4:** Experimental MDOF Mode 1 estimates: (a) nominal linear FRF (solid line: low-level 'linear' FRF; bold dashed line: nominal linear FRF estimate), (b) cubic nonlinear coefficients (bold solid line: $A_1^{1}(1,1,1)$; bold dashed line: $A_1^{1}(1,1,2)$; solid line: $A_1^{1}(1,2,2)$; dashed line: $A_1^{1}(2,2,2)$).
3.2 Experimental MDOF Nonlinear Parameter Estimation

The true value in the proposed identification method is the ability to conduct predictions based upon the assembled modal model. Fig. 7 displays the results of such a prediction, as well as the values of the identified coefficients. In Fig. 7, the assembled nonlinear reduced order model is compared with experimental data representing twice the load (8 versus 4g RMS) used for the identification. The response PSDs of the beam midpoint displayed in Fig. 7 equate to RMS values of the estimate and experiment beam midpoint displacement of 0.045in versus 0.047in respectively. The spectral comparisons are also quite good, particularly for the first mode. Note the appearance of a peak in the 200 Hz range of the 'estimated' PSD. It is not clear whether the unknown peak is an artifact of the numerical integration, or one associated with the identification itself, i.e., the most appropriate model given the influence of the sensors. As previously mentioned, an issue under investigation is the number of sensors and their appropriate positioning. Recall that for this test, the accelerometers were positioned on only one side of the beam. It is theorized that the asymmetrical location of the sensors resulted in the excitation of an asymmetric mode not included in the basis set. Further, the modal mass of the beam relative the sensor mass will also play a significant role for such a lightly damped, light-weight structure. An optimal testing configuration would be laser vibrometers appropriately spaced along the beam, but this was not a viable option at the time of testing.
Figure 6: Experimental prediction of 8g RMS loading using identified nonlinear coefficients.  
$A_1^{(1,1,1)} = 1.27e+8 \text{ in}^2\text{s}^{-2}$; $A_1^{(1,1,2)} = 4.12e+8 \text{ in}^2\text{s}^{-2}$; $A_1^{(1,2,2)} = 1.38e+9 \text{ in}^2\text{s}^{-2}$; $A_1^{(2,2,2)} = 4.02e+9 \text{ in}^2\text{s}^{-2}$; $A_2^{(1,1,1)} = 1.49e+8 \text{ in}^2\text{s}^{-2}$; $A_2^{(1,1,2)} = 1.27e+9 \text{ in}^2\text{s}^{-2}$; $A_2^{(1,2,2)} = 4.72e+9 \text{ in}^2\text{s}^{-2}$; $A_2^{(2,2,2)} = 1.35e+10 \text{ in}^2\text{s}^{-2}$.  (Dashed line: Experimental spectrum; Bold solid line: Prediction).

4. Conclusion and Future Work

The modal NIFO method was presented for the estimation and generation of nonlinear modal models using experimental data. Accurate estimates were obtained for an MDOF experimental modal model. Good RMS and spectral comparisons were made between experimental and the modal NIFO based reduced order model, at a significantly higher input scenario. The method presented in this study is quite flexible in that it is a simple matter to adjust the assumed form of the nonlinearity. As the end result is a reduced order model, researchers have the ability to quickly and accurately predict the design space. Future work will investigate the issue of spatial truncation in developing experimental modal vectors, as well as the issues associated with closely spaced modes. It is expected that the developing tool will be of great benefit for the analysis of highly complex structures in extreme combined environments.

Reference:


